

What is combinatorics?

January 21, 2016

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- ▶ What is combinatorics anyway?

Combinatorics subjects

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- ▶ Is the arrangement possible? (Existence)
- ▶ In how many ways can the arrangement be made? (Enumeration)
- ▶ How do we go about finding such an arrangement? (Design and algorithm)

Example 1: Sudoku

Fill a 9×9 grid with digits so that each column, each row, and each of the nine 3×3 sub-grids contains all of the digits from 1 to 9:

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Example 2: Seven Bridge Problem

The city of Königsberg in Prussia (now Kaliningrad, Russia) was set on both sides of the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges.

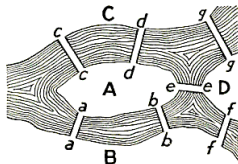
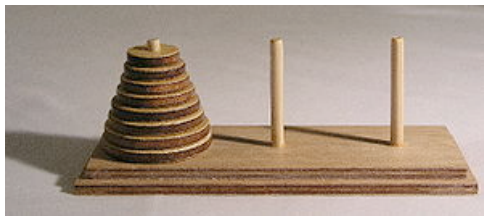


FIGURE 98. *Geographic Map:
The Königsberg Bridges.*

The problem was to find a walk through the city that would cross each bridge once and only once. The islands could not be reached by any route other than the bridges, and every bridge must have been crossed completely every time.

Example 3: Hanoi Tower Puzzle



The objective of the puzzle is to move the entire stack to another rod, obeying the following rules:

- ▶ Only one disk may be moved at a time.
- ▶ Each move consists of taking the upper disk from one of the rods and sliding it onto another rod, on top of the other disks that may already be present on that rod.
- ▶ No disk may be placed on top of a smaller disk.

Example 4: Euler's officers

Six different regiments have six officers, each one holding a different rank (of six different ranks altogether). Can these 36 officers be arranged in a square formation so that each row and column contains one officer of each rank and one from each regiment? (Euler, 1782)

			?	?	?
			?	?	?
			?	?	?
?	?	?	?	?	?
?	?	?	?	?	?
?	?	?	?	?	?

Example 5: Kirkman's schoolgirls

Fifteen schoolgirls go for a walk every day for a week in five rows of three. Is it possible to arrange the walks so that every two girls walk together exactly once during the week? (Thomas Kirkman, 1843)



Example 6: Mapping coloring

For any given map, you can always use 4 colors to color the states so that states sharing a boundary have different colors?(Francis Guthrie, 1852)



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- ▶ Extremal set theory (hypergraphs) and partial order sets